Investigation of the inherent trade-off between bias model complexity and state estimation accuracy in INS/GNSS-Integration

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Objectives of the investigation

- INS/GNSS integration forms the central navigation unit for many outdoor applications
- Knowledge of IMU sensor errors
- Impact of accelerometer bias modelling on the navigation solution accuracy
Outline

1. Analysis of long-term IMU recordings
2. Modelling of IMU errors in a loose INS/GNSS integration architecture
3. Simulation study and results
4. Conclusions and Outlook
Analysis of long-term IMU recordings

- Analysis based on the Allan Variance Method
- Goal: Identification and quantification of the underlying noise processes
- Analysis of a tactical grade IMU (3-axis servo-accelerometer & 3-axis fiber optic gyro)
Allan Variance – Basic concept

1. Data record length: 6 hours (non-moving IMU)

2. Compute the average of each block (n … number of blocks)
   List of averages \([\bar{u}(\tau)_1 \quad \bar{u}(\tau)_2 \quad \cdots \quad \bar{u}(\tau)_n]\)

3. Allan variance \([1]\)
   \[
   \sigma^2(\tau) = \frac{1}{2(n-1)} \sum_{i}^{n-1} (\bar{u}(\tau)_{i+1} - \bar{u}(\tau)_i)^2
   \]
   Allan deviation: \(\sigma(\tau) = \sqrt{\sigma^2(\tau)}\)
Allan Variance – Results I

\[ z_g(t) = z_{g,N}(t) \]
White noise

\[ z_a(t) = z_{a,A}(t) + z_{a,N}(t) + z_{a,B}(t) + z_{a,K}(t) \]
Quantization noise White noise Flicker noise Random walk
Allan Variance – Results II

- Estimating the parameters of the noise processes via a LSQ fit
- Defining the parameter vector $\vartheta$
  $$\vartheta = [S_{a,A} \quad S_{a,N} \quad S_{a,B} \quad T_B \quad S_{a,K}]$$
- First order Gauß Markov (FOGM) process is used to approximate the flicker noise process
Modelling of IMU errors in a loose INS/GNSS integration architecture

- IMU observation model

\[ \tilde{f} = f + b_a + z_{a,N} \]
\[ \tilde{\omega} = \omega + b_g + z_{g,N} \]

- IMU bias model

\[ b_a = b_{a,0} + z_{a,B} + z_{a,K} \]
\[ b_g = b_{g,0} \]

static biases
Modelling of IMU errors in a loose INS/GNSS integration architecture

- Structure of the classical system model [4]:

\[
\begin{bmatrix}
\delta \psi_m \\
\delta \theta_m \\
\delta \phi_m \\
\delta b_a \\
\delta b_g
\end{bmatrix} =
\begin{bmatrix}
F_{\psi \psi} & F_{\psi \theta} & F_{\psi \phi} & 0_3 & C^m_b \\
F_{\theta \psi} & F_{\theta \theta} & F_{\theta \phi} & C^m_b & 0_3 \\
F_{\phi \psi} & F_{\phi \theta} & F_{\phi \phi} & 0_3 & 0_3 \\
0_3 & 0_3 & 0_3 & 0_3 & 0_3 \\
0_3 & 0_3 & 0_3 & 0_3 & 0_3
\end{bmatrix}
\begin{bmatrix}
\delta \psi_m \\
\delta \theta_m \\
\delta \phi_m \\
\delta b_a \\
\delta b_g
\end{bmatrix} +
\begin{bmatrix}
C^m_b \\
0_3 \\
C^m_b \\
0_3 & 0_3 & 0_3 \\
0_3 & 0_3 & 0_3
\end{bmatrix}
\begin{bmatrix}
w_{g,N} \\
w_{a,N}
\end{bmatrix}
\]

- Structure of the detailed system model:

\[
\begin{bmatrix}
\delta \psi^n \\
\delta \theta^n \\
\delta \phi^n \\
\delta b_a \\
\delta b_g
\end{bmatrix} =
\begin{bmatrix}
F_{\psi \psi} & F_{\psi \theta} & F_{\psi \phi} & 0_3 & C^n_b \\
F_{\theta \psi} & F_{\theta \theta} & F_{\theta \phi} & C^n_b & 0_3 \\
F_{\phi \psi} & F_{\phi \theta} & F_{\phi \phi} & 0_3 & 0_3 \\
0_3 & 0_3 & 0_3 & 0_3 & 0_3 \\
0_3 & 0_3 & 0_3 & 0_3 & 0_3
\end{bmatrix}
\begin{bmatrix}
\delta \psi^n \\
\delta \theta^n \\
\delta \phi^n \\
\delta b_a \\
\delta b_g
\end{bmatrix} +
\begin{bmatrix}
C^n_b \\
0_3 & 0_3 & 0_3 \\
0_3 & C^n_b & 0_3 & 0_3 \\
0_3 & 0_3 & 0_3 & 0_3 \\
0_3 & 0_3 & I_3 & I_3 \\
0_3 & 0_3 & 0_3 & 0_3
\end{bmatrix}
\begin{bmatrix}
w_{g,N} \\
w_{a,N} \\
w_{a,B} \\
w_{a,K}
\end{bmatrix}
\]
Modelling of IMU errors in a loose INS/GNSS integration architecture

- Classical system noise VCM:

\[ Q_{k-1} = T_{k-1} G_{k-1} \begin{bmatrix} I_3 S_{g,N} \\ I_3 S_{a,N} \end{bmatrix} G_{k-1}^T T_{k-1}^T \Delta t \]

- Detailed system noise VCM:

\[ Q'_{k-1} = T'_{k-1} G'_{k-1} \begin{bmatrix} I_3 S_{g,N} \\ I_3 S_{a,N} \\ I_3 S_{a,B} \\ I_3 S_{a,K} \end{bmatrix} G'^T_{k-1} T'^T_{k-1} \Delta t \]
Simulation study

1. Classical modeling approach via WN processes (N model)
   - Applying the manufacturer WN specification
2. Detailed modeling approach via WN, FOGM and RW (NBK model)
   - Applying the estimated noise parameters

The two modeling approaches are compared for two cases:
1. Continuous GNSS coverage (1 Hz)
2. GNSS signal outage over a period of 5 minutes
Simulation study – Motion scenario

- True IMU and GNSS observations are determined from the simulated motion scenario.
- Generate sensor errors:
  - RTK precision for the GNSS observation errors.
  - Replicate an IMU with identical stochastic properties as those of the IMU investigated (except quantization noise).
Simulation results – Case 1 (No GNSS outages)

White noise of NBK model: \(8.2 \mu g/\sqrt{Hz}\)
White noise of N model: \(50 \mu g/\sqrt{Hz}\)
Simulation results – Case 1 (No GNSS outages)

White noise of NBK model: 8.2 μg/√Hz
White noise of N model: 50 μg/√Hz
Simulation results – Case 2 (GNSS outages for 5 min)
Simulation results – Case 2 (GNSS outages for 5 min)
Conclusions

- Successful identification and quantification of the different noise processes for the investigated IMU
- Incorporation of a detailed accelerometer bias model into the loose INS/GNSS integration architecture
  - Additional research to include quantization noise
- The largest contribution to the accuracy of the navigation solution came from errors in the gyros and not from errors in the accelerometers
Outlook

• Solely estimated standard deviations were investigated
  – Investigation of true errors is possible in case of simulation studies
• Environmental induced errors are not taken into account by the AV method, but are frequently encountered in practice
  – Vibrations, temperature changes
• The conducted investigations will be verified on real world applications
  – Coverage of a wide range of vehicle dynamics
References


[2] StackExchange, How to interpret Allan Deviation plot for gyroscope?


Appendix I
Appendix II.a
Appendix II.b
Appendix III
**Appendix V**

**Tab. 1: Estimated noise parameters of the x-axis accelerometer and the x-axis gyro**

<table>
<thead>
<tr>
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</tr>
</thead>
<tbody>
<tr>
<td>Quantization noise</td>
<td>$S_A \left[ m^2 s^{-2} \right]$</td>
<td>$1.4841 e^{-5}$</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>$A \left[ ms^{-1} \right]$</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>White noise</td>
<td>$S_N \left[ m^2 s^{-3} \right], \left[ rad^2 s^{-1} \right]$</td>
<td>$6.2865 e^{-9}$</td>
<td>-</td>
<td>$5.3079 e^{-10}$</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>$N \left[ \mu g/\sqrt{Hz} \right], \left[ deg/\sqrt{h} \right]$</td>
<td>$8.1$</td>
<td>$&lt; 50$</td>
<td>$0.08$</td>
<td>$&lt; 0.15$</td>
</tr>
<tr>
<td>Flicker noise</td>
<td>$S_B \left[ m^2 s^{-5} \right]$</td>
<td>$2.4360 e^{-11}$</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>$B \left[ \mu g \right], \left[ deg/h \right]$</td>
<td>$2.2$</td>
<td>$&lt; 10$</td>
<td>-</td>
<td>$&lt; 0.1$</td>
</tr>
<tr>
<td></td>
<td>$T_B \left[ s \right]$</td>
<td>$45$</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Random walk</td>
<td>$S_K \left[ m^2 s^{-5} \right]$</td>
<td>$2.6731 e^{-12}$</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>$K \left[ ms^{-5/2} \right]$</td>
<td>$1.6350 e^{-6}$</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>
Appendix VI

- Overall PSD of the stochastic accelerometer errors (Superposition principle)

\[ S_z(f) = S_{z,A}(f) + S_{z,N}(f) + S_{z,B}(f) + S_{z,K}(f) \]

- AV is related to the PSD via

\[ \sigma_z^2(\tau) = 4 \int_0^\infty S_z(f) \frac{\sin^4(\pi f \tau)}{(\pi f \tau)^2} df \]

- Overall AV for the accelerometers

\[ \sigma_z^2(\tau) = \sigma_{z,A}^2(\tau) + \sigma_{z,N}^2(\tau) + \sigma_{z,B}^2(\tau) + \sigma_{z,K}^2(\tau) \]

\[ = \frac{3S_{z,A}}{\tau^2} + \frac{S_{z,N}}{\tau} + \frac{S_{z,B}T_B^2}{\tau} \left[ 1 - \frac{T_B}{2\tau} \left( 3 - 4e^{-\frac{\tau}{T_B}} + 4e^{-\frac{2\tau}{T_B}} \right) \right] + \frac{S_{z,K}}{3\tau} \]

- Defining the parameter vector \( \varTheta \)

\[ \varTheta = [S_{z,A} \quad S_{z,N} \quad S_{z,A} \quad T_B \quad S_{z,K}] \]